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Holography of Charged Dilaton Black Holes in General Dimensions

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Abstract

We study several aspects of charged dilaton black holes with planar symmetry in $(d+2)$-dimensional spacetime, generalizing the four-dimensional results investigated in arXiv:0911.3586 [hep-th]. We revisit the exact solutions with both zero and finite temperature and discuss the thermodynamics of the near-extremal black holes. We calculate the AC conductivity in the zero-temperature background by solving the corresponding Schrödinger equation and find that the AC conductivity behaves like ω^δ, where the exponent δ is determined by the dilaton coupling α and the spacetime dimension parameter d. Moreover, we also study the Gauss-Bonnet corrections to η/s in a five-dimensional finite-temperature background.

Contents

1 Introduction

The AdS/CFT correspondence [1, 2] has been shown to be a useful tool for studying the dynamics of a strongly coupled field theory, since many physical properties of field theory can be derived by its weakly coupled and tractable dual gravitational description. In recent years numerous applications for the AdS/CFT correspondence, such as in QCD etc., have been explored in detail. In particular, the investigation of certain condensed matter systems, namely the AdS/CMP correspondence, has accelerated quickly in the past years. Some excellent reviews have appeared [3].

In order to study the gravity dual of a condensed matter system at finite temperature, we

need to consider a suitably corresponding black hole for the background spacetimes. One particular class of interesting backgrounds is the charged dilaton black holes [4, 5, 6, 7] of the Einstein-Maxwell-dilaton theory in which the dilaton field is exponentially coupled with the gauge field, $e^{2\alpha\phi}F^2$. A specific property of this type of black hole is that the Bekenstein-Hawking entropy vanishes at the extremal limit for any value of the non-zero dilaton coupling, $\alpha \neq 0$, therefore the higher curvature corrections are crucial. In addition, the charged dilaton black holes with a Liouville potential were studied, e.g. in [8], and the results suggest that their AdS generalizations may provide interesting holographic descriptions of certain condensed matter systems.

Recently, the holography of charged dilaton black holes in AdS_4 with planar symmetry was extensively investigated in [9]. It turns out that the near horizon geometry was Lifshitz-like with a dynamical exponent z determined by the dilaton coupling. The global solution was constructed via numerical methods, and the attractor behavior was also discussed. The authors also examined the thermodynamics of near extremal black holes and computed the AC conductivity in a zero-temperature background. For related work on charged dilaton black holes see [10, 11, 12, 13].

In this paper we generalize the work of [9] in four dimensional spacetime to arbitrary $(d+2)$-dimensions. For a practical application to a specific system in condensed matter physics, the value of d is given. For example, one should choose $d = 2$ to study the $(2+1)$-dimensional layered systems. However, our physical spacetime may be higher dimensional with tiny extra dimensions, and more spacetime dimensions might be holographically generated when extra adjoint fields are involved. Therefore, it is of interest to consider the generalization to various dimensions and try to find the universal behavior.

By considering a $(d+2)$-dimensional Einstein-Maxwell-dilaton action with dilaton coupling of the form $e^{2\alpha\phi}$, in both zero and finite temperatures, we obtain particular exact scaling solutions which are expected to be the near horizon geometries of the considered black hole backgrounds in AdS_{d+2}. The zero-temperature solution is still Lifshitz-like and the dynamical exponent is determined by α and the spacetime dimension. The thermodynamics of the near extremal black holes is also studied. Furthermore, we compute the AC conductivity in the zero-temperature background. We can transform the corresponding equation of motion into a Schrödinger form with an effective potential of the form $V(z) = c/z^2$ and then determine the frequency dependence of the AC conductivity

as $\mathrm{Re}(\sigma) \sim \omega^{\delta}$. Here both constants c and δ are determined by α and d. Moreover, we compute the Gauss-Bonnet corrections to η/s in a five-dimensional finite temperature background, and the result agrees with the well-known AdS counterpart when the dynamical exponent $z \to 1$.

The rest of the paper is organized as follows. In Section 2 we present the exact solutions in $(d+2)$-dimensional spacetime. The thermodynamics of the near extremal black holes is discussed in Section 3, and the AC conductivity is calculated in Section 4. The derivation of the Gauss-Bonnet corrections to η/s in a five-dimensional finite temperature background is given in Section 5. A summary and discussion will be given in the final part.

2 The solution

In this section we will exhibit the exact solutions, including both zero and finite temperature cases. Consider the following Einstein-Maxwell-dilaton action in $(d+2)$-dimensional spacetime with a negative cosmological constant:

$$S = \frac{1}{16\pi G_{d+2}} \int d^{d+2}x\sqrt{-g}\left(R - 2\Lambda - 2\partial_\mu\phi\partial^\mu\phi - \mathrm{e}^{2\alpha\phi}F_{\mu\nu}F^{\mu\nu}\right), \tag{2.1}$$

the corresponding equations of motion can be summarized as follows:

$$\begin{aligned} &R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(\partial\phi)^2 + 2\mathrm{e}^{2\alpha\phi}F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{2}\mathrm{e}^{2\alpha\phi}g_{\mu\nu}F^2, \\ &\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) = \frac{\alpha}{2}\sqrt{-g}\mathrm{e}^{2\alpha\phi}F^2, \qquad \partial_\mu\left(\sqrt{-g}\mathrm{e}^{2\alpha\phi}F^{\mu\nu}\right) = 0. \end{aligned} \tag{2.2}$$

The cosmological constant is related to the AdS radius L by

$$\Lambda = -\frac{d(d+1)}{2L^2}, \tag{2.3}$$

and we will set $L = 1$ for simplicity in the following. The dependence on L can be recovered simply by a dimensional analysis.

Let us take the following ansatz for the planar symmetric metric and gauge potential:

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{a^2(r)} + b^2(r)\sum_{i=1}^{d} dx_i^2, \qquad A = A_t(r)dt, \tag{2.4}$$

then the solution for the gauge field is

$$F_{tr} = q_e b^{-d} \mathrm{e}^{-2\alpha\phi}, \tag{2.5}$$

and the other equations of motion can be reformulated as

$$\begin{aligned}
\phi'^2 + \frac{d}{2}\frac{b''}{b} &= 0, && (2.6)\\
\left(a^2 b^{2d-2}\right)'' - 2(d-2)ab^{d-2}b'\left(ab^{d-1}\right)' + 4\Lambda b^{2d-2} &= 0, && (2.7)\\
\left(a^2 b^d \phi'\right)' + \alpha q_e^2 b^{-d} \mathrm{e}^{-2\alpha\phi} &= 0, && (2.8)
\end{aligned}$$

together with the first order constraint coming from the rr-component of the Einstein equation

$$daa'bb' + \frac{1}{2}d(d-1)a^2 b'^2 + \Lambda b^2 = \phi'^2 a^2 b^2 - q_e^2 b^{-2d+2} \mathrm{e}^{-2\alpha\phi}. \tag{2.9}$$

In order to find the near-horizon solution, we define the new variable $w = r - r_H$ where r_H denotes the radius of the horizon, and the scaling solution should be like

$$a(w) = a_0 w^\gamma, \qquad b(w) = b_0 w^\beta, \qquad \phi(w) = -k_0 \ln w, \tag{2.10}$$

where a_0, b_0 and k_0 are constants. After some algebra we can find the following zero-temperature solution (fixing $b_0 = 1$ by rescaling x_i):

$$\begin{aligned}
&\gamma = 1, \qquad \beta = \frac{\alpha^2}{\alpha^2 + 2d},\\
&a_0^2 = -\frac{2\Lambda}{(d\beta+1)(d\beta-\beta+1)}, \quad q_e^2 = -\frac{2\Lambda}{\alpha^2+2}, \quad k_0 = \frac{\alpha d^2}{2(\alpha^2+2d)}.
\end{aligned} \tag{2.11}$$

The corresponding finite-temperature solution can be easily generalized,

$$ds^2 = -a^2(w)f(w)dt^2 + \frac{dw^2}{a^2(w)f(w)} + b^2(w)\sum_{i=1}^{d} dx_i^2, \qquad f(w) = 1 - \frac{w_0^{d\beta+1}}{w^{d\beta+1}}, \tag{2.12}$$

with the other fields and parameters remaining invariant.

Here are some comments on these solutions:

- The zero temperature solutions with Lifshitz-like scaling symmetry has already been obtained in [14], and the above solutions reduce to those obtained in [9] when $d = 2$.

- The near-horizon metric takes a Lifshitz-like form with anisotropic scaling [15] whose dynamical exponent is $z = 1/\beta$. However, it cannot be treated as the genuine gravity dual of the Lifshitz fixed-point, since the scaling symmetry is broken by the non-trivial dilaton.[1]

- Following the spirit of [9], here we are interested in finding asymptotically AdS_{d+2} solutions, the near horizon geometries of which are either (2.10) for zero temperature or (2.12) for finite temperature (both are exact solutions). From (2.11) we can see that the charge parameter q_e is fixed, while in the asymptotically AdS_{d+2} case q_e is related to the number density in the dual field theory.

3 Thermodynamics

We will discuss the thermodynamics of the finite-temperature solution in this section. First we recall the finite-temperature solution,

$$ds^2 = -a_0^2 w^2 f(w) dt^2 + \frac{dw^2}{a_0^2 w^2 f(w)} + w^{2\beta} \sum_{i=1}^{d} dx_i^2, \qquad f(w) = 1 - \frac{w_0^{d\beta+1}}{w^{d\beta+1}}. \tag{3.1}$$

As $w \to \infty$, this solution reduces to the original scaling solution. Since the scaling solution (2.11) corresponds to the near horizon of an extremal black hole, it can be expected that the finite-temperature solution corresponds to the near-horizon region of a near-extremal black hole.

The temperature of the black hole is given by

$$T = \frac{(\beta d + 1) a_0^2}{4\pi} w_0, \tag{3.2}$$

and the entropy density is

$$s = \frac{1}{4} b^d(w)\Big|_{w=w_0} = \frac{1}{4} w_0^{\beta d} \sim T^{\beta d}. \tag{3.3}$$

For a charged black hole, the entropy density can be expressed as a function of the temperature T and the chemical potential μ. Since the dimensions of T and μ are

[1] Related work on Lifshitz black holes is listed in [16]

dim T = dim $\mu = [M]$, the entropy density of a slightly non-extremal black hole in $(d+2)$-dimensions is

$$s \sim T^{\beta d} \mu^{d-\beta d} \tag{3.4}$$

by dimensional analysis. The entropy density can also be obtained by the standard Euclidean path integral, which gives

$$s = aCT^{\beta d} \mu^{d-\beta d}, \qquad C \sim L^d / G_{d+2}. \tag{3.5}$$

Here G_{d+2} denotes the $(d+2)$-dimensional Newton constant, and the coefficient a depends on α and the asymptotic value of the dilaton ϕ_0. Here the specific heat,

$$C_v = T\left(\frac{ds}{dT}\right)_\mu = aC\beta d T^{\beta d} \mu^{d-\beta d}, \tag{3.6}$$

is always positive. The other thermodynamical quantities can be obtained by the entropy density via the Gibbs-Duhem relation $sdT - dP + nd\mu = 0$. Here P and n are the pressure and number density. Keeping μ fixed and performing the integration gives

$$P = \int sdT = \frac{a}{\beta d + 1} C \mu^{d-\beta d} T^{\beta d + 1} + P_0(\mu), \tag{3.7}$$

where $P_0(\mu)$ is a temperature independent integration constant which can be fixed by dimensional analysis:

$$P_0(\mu) = bC\mathrm{e}^{(d+1)\alpha\phi_0} \mu^{d+1}. \tag{3.8}$$

Then from the relation

$$dP = aC\mu^{d-\beta d} T^{\beta d} dT + \frac{(d-\beta d)}{\beta d + 1} aC\mu^{d-1-\beta d} T^{\beta d+1} d\mu + bC(d+1)\mathrm{e}^{(d+1)\alpha\phi_0} \mu^d d\mu, \tag{3.9}$$

one can identify the number density n as

$$n = \frac{(d-\beta d)}{\beta d + 1} aC\mu^{d-1-\beta d} T^{\beta d+1} + bC(d+1)\mathrm{e}^{(d+1)\alpha\phi_0} \mu^d. \tag{3.10}$$

Finally, the energy density is determined by the relation $\rho = Ts + \mu n - P$, which gives

$$\rho = \frac{d}{\beta d + 1} aC\mu^{d-\beta d} T^{\beta d+1} + dbC\mathrm{e}^{(d+1)\alpha\phi_0} \mu^{d+1}. \tag{3.11}$$

The equation of state of this near-extremal system is

$$P = \frac{1}{d}\rho. \tag{3.12}$$

The susceptibility is given by

$$\chi \equiv \left(\frac{\partial n}{\partial \mu}\right)_T = \frac{(d-1-\beta d)(d-\beta d)}{\beta d+1} aCT^{\beta d+1}\mu^{d-2-\beta d} + d(d+1)bC\mathrm{e}^{(d+1)\alpha\phi_0}\mu^{d-1}, \quad (3.13)$$

which is positive when $T \ll \mu$. Notice that the first term becomes negative when $d-1-\beta d < 0$, i.e. $\beta > (d-1)/d$. As emphasized in [9], the formulae in this section are valid when $T \ll \mu$. Furthermore, whether χ changes sign as the temperature increases, signalling a phase transition, requires one to explore beyond the regime $T \ll \mu$.

Rather than constructing the global solution which is asymptotically AdS_{d+2}, we focus on some qualitative behavior of the asymptotic solution. Similar to the four-dimensional example [9], the bulk solutions related by a rescaling of coordinates should be treated as being distinct with different chemical potential. Therefore, all solutions can be obtained by a suitable rescaling and shift in the dilaton from a particular simple solution, e.g. $q_e = 1$ and $\phi_0 = 0$. The rescaling is given by

$$r \to \lambda r, \qquad (t, x_i) \to \lambda^{-1}(t, x_i). \quad (3.14)$$

Furthermore, from the equations of motion we can see that the metric and $\phi - \phi_0$ only depend on $q_e^2 \mathrm{e}^{-2\alpha\phi_0}$.

Reconsidering the equations of motion (2.6)–(2.8) and the constraint (2.9), we can see that for an asymptotically AdS_{d+2} solution, the metric and dilaton must take the following form:

$$\begin{aligned} a^2(r) &= r^2\left(1 - e_1\frac{\rho}{r^{d+1}} + \frac{q_e^2\mathrm{e}^{-2\alpha\phi_0}}{r^{2d}} + \cdots\right), \\ b^2(r) &= r^2\left(1+\cdots\right), \\ \phi &= \phi_0 + \frac{\phi_1}{r^{d+1}} + \cdots, \end{aligned} \quad (3.15)$$

where the ellipses denote terms that are subdominant at large r. The parameter ρ is the energy density of the black hole, and e_1 is a constant depending on L. Under the rescaling of (3.14), the corresponding rescaling of the energy density and charge parameter should be

$$\rho \to \lambda^{d+1}\rho, \qquad q_e \to \lambda^d q_e. \quad (3.16)$$

This implies the following relation:

$$\rho = D_1\left(q_e\mathrm{e}^{-\alpha\phi_0}\right)^{\frac{d+1}{d}}, \quad (3.17)$$

where D_1 is an α dependent parameter. Similarly, the chemical potential,

$$\mu = \int_{r_h}^{\infty} \frac{q_e}{b^d(r)} \mathrm{e}^{-2\alpha\phi} dr, \tag{3.18}$$

can be determined by a similar rescaling argument as

$$\mu = D_2 \left(q_e \mathrm{e}^{-\alpha\phi_0}\right)^{\frac{1}{d}} \mathrm{e}^{-\alpha\phi_0}, \tag{3.19}$$

where D_2 is also an α dependent parameter. This gives

$$\rho = D_3 \mathrm{e}^{(d+1)\alpha\phi_0} \mu^{d+1}, \tag{3.20}$$

which agrees with the second term in (3.11).

4 Conductivity in zero-temperature backgrounds

We will calculate the conductivity σ in the $(d+2)$-dimensional extremal black hole background, generalizing the result obtained in [9]. A useful formulation was proposed in [17], which stated that after introducing a perturbative gauge field $A_x(r,t)$, the corresponding equation of motion for A_x could be recast in a Schrödinger-like form

$$-A_{x,zz} + V(z) A_x = \omega^2 A_x, \tag{4.1}$$

where z is a redefinition of the radial variable r. Then by studying scattering with ingoing boundary condition at the horizon, the conductivity is determined in terms of the reflection coefficient

$$\sigma(\omega) = \frac{1-\mathcal{R}}{1+\mathcal{R}}. \tag{4.2}$$

It has been pointed out in [17] that such a formulation could be generalized to higher-dimensional cases.

In the following we will perform the calculations in our $(d+2)$-dimensional extremal background. Our task is still to find the equation for A_x and cast it in the form of (4.1). Consider the general metric of the form

$$ds^2 = -g(r)\mathrm{e}^{-\chi(r)} dt^2 + \frac{dr^2}{g(r)} + r^2 \sum_{i=1}^{d} dx_i^2, \tag{4.3}$$

which is extensively adopted in the discussions of holographic superconductors [18]. The gauge field, including the perturbation, is given by

$$A = A_t(r)dt + \tilde{A}_x(r)\mathrm{e}^{-i\omega t}dx. \tag{4.4}$$

Consider the Lagrangian of the following form:

$$\mathcal{L} = \cdots - 2(\nabla\phi)^2 - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu} + \cdots\,, \tag{4.5}$$

where the gauge coupling is $f^2(\phi)$. The t-component of the Maxwell equation determines the background A_t,

$$\partial_r A_t = q_e f^{-2}(\phi) r^{-d}\mathrm{e}^{-\chi/2}, \tag{4.6}$$

and the x-component equation,

$$\begin{aligned}\omega^2 f^2(\phi) r^{d-2} g^{-1}\mathrm{e}^{\chi/2}\tilde{A}_x + \partial_r\left(f^2(\phi)r^{d-2}\mathrm{e}^{-\chi/2}g\partial_r\tilde{A}_x\right)&\\ +f^2(\phi)r^{d-2}\mathrm{e}^{\chi/2}\partial_r A_t\left(\partial_r\tilde{g}_{tx} - \frac{2}{r}\tilde{g}_{tx}\right) &= 0.\end{aligned} \tag{4.7}$$

Notice that g_{tx} should be turned on at the same order as the gauge field perturbation and we denote $g_{tx}(t,r) = \mathrm{e}^{-i\omega t}\tilde{g}_{tx}(r)$. Furthermore, the rx-component of the Einstein equations gives

$$\partial_r\tilde{g}_{tx} - \frac{2}{r}\tilde{g}_{tx} = -f^2(\phi)\tilde{A}_x\partial_r A_t. \tag{4.8}$$

Substituting (4.8) into (4.7), we can obtain

$$\partial_r\left(f^2(\phi)r^{d-2}g\mathrm{e}^{-\chi/2}\partial_r\tilde{A}_x\right) + \omega^2 f^2(\phi)r^{d-2}g^{-1}\mathrm{e}^{\chi/2}\tilde{A}_x - f^4(\phi)r^{d-2}\mathrm{e}^{\chi/2}(\partial_r A_t)^2\tilde{A}_x = 0, \tag{4.9}$$

which agrees with (3.7) of [9] for $d = 2$. The background $\partial_r A_t$ is given by (4.6). By taking a new coordinate z and a new wavefunction Ψ,

$$\partial_z = \mathrm{e}^{-\chi/2}g\,\partial_r, \qquad \Psi = f(\phi)r^{\frac{d-2}{2}}\tilde{A}_x. \tag{4.10}$$

this equation becomes a Schrödinger equation:

$$-\Psi'' + V(z)\Psi = \omega^2\Psi, \tag{4.11}$$

where the potential is

$$V(z) = f^{-1}(\phi)r^{-\frac{d-2}{2}}\partial_z^2\left(f(\phi)r^{\frac{d-2}{2}}\right) + q_e^2 f^{-2}(\phi)r^{-2d}g\mathrm{e}^{-\chi}. \tag{4.12}$$

Here prime stands for the derivative with respect to z, and it agrees with (3.15) of [9] once again when $d = 2$.

4.1 Near horizon analysis

Recall the metric of the zero-temperature background,

$$ds^2 = -a_0^2 w^2 dt^2 + \frac{dw^2}{a_0^2 w^2} + w^{2\beta} \sum_{i=1}^{d} dx_i^2. \tag{4.13}$$

Comparing with (4.3), we can obtain

$$r = w^\beta, \qquad g(r) = \beta^2 a_0^2 r^2, \qquad \mathrm{e}^{-\chi(r)} = \frac{1}{\beta^2} r^{\frac{2}{\beta}-2}. \tag{4.14}$$

Then

$$\frac{\partial r}{\partial z} = \mathrm{e}^{-\chi/2} g = \beta a_0^2 r^{\frac{1}{\beta}+1} \quad \Rightarrow \quad z = -\frac{1}{a_0^2 w}. \tag{4.15}$$

Here the gauge coupling is

$$f(\phi) = 2\exp(\alpha\phi) = \frac{2}{w^{\beta d}}. \tag{4.16}$$

Plugging (4.15) and (4.16) into (4.12), we obtain the following expression for $V(z)$

$$V_0(z) = \frac{c_0}{z^2}, \tag{4.17}$$

where

$$c_0 = \frac{(d+2)^2}{4}\beta^2 - \frac{d+2}{2}\beta + \frac{q_e^2}{4a_0^2}. \tag{4.18}$$

It can be seen that for a general $(d+2)$-dimensional extremal charged dilaton black holes, the equation of the gauge field perturbation $\tilde{A}_x$ can always be transformed into a Schrödinger equation. Furthermore, the effective potential takes a universal form $V_0(z) = c_0/z^2$, where c_0 is a constant which is determined by α and d.

The technique of solving the Schrödinger equation with specific potential is summarized in the Appendix. The ingoing mode solution is

$$\Psi(z) = C_0^{(\mathrm{in})} \sqrt{-\frac{\pi\omega z}{2}} H_{\nu_0}^{(1)}(-\omega z) \sim C_0^{(\mathrm{in})} \mathrm{e}^{-i(\omega z + \frac{1}{2}\nu_0\pi + \frac{1}{2}\pi)}, \tag{4.19}$$

where $\nu_0^2 = c_0 + 1/4$.

4.2 Asymptotic analysis

For the asymptotic solution we have

$$\chi = 0, \qquad g = r^2, \qquad f = f(\phi_0), \tag{4.20}$$

so

$$\frac{\partial r}{\partial z} = r^2 \quad \Rightarrow \quad z = -\frac{1}{r}. \tag{4.21}$$

Unlike the $d = 2$ case in [9], a crucial difference is that the effective potential cannot be neglected at the asymptotic boundary:

$$V_\infty(z) = \frac{c_\infty}{z^2}, \qquad c_\infty = \frac{d(d-2)}{4}. \tag{4.22}$$

The general solution is (refer to the Appendix for the details)

$$\begin{aligned} \Psi(z) &= \frac{\pi}{\Gamma(\nu_\infty)}\sqrt{-\omega z}\left(C_\infty^{(1)} H_{\nu_\infty}^{(1)}(-\omega z) + C_\infty^{(2)} H_{\nu_\infty}^{(2)}(-\omega z)\right) \\ &\to -i\left(C_\infty^{(1)} - C_\infty^{(2)}\right)\sqrt{-\omega z}\left(-\frac{2}{\omega z}\right)^{\nu_\infty}, \end{aligned} \tag{4.23}$$

where $\nu_\infty = \sqrt{c_\infty + 1/4} = (d-1)/2$.

4.3 Matching

In order to match the coefficients in the near horizon and asymptotic analysis, we should take the small ω limit to extrapolate the near horizon and asymptotic wavefunctions to an intermediate region of small $-\omega z$. From the near horizon side ($-z \gg 1$), we have

$$\Psi(z) = C_0^{(\text{in})}\sqrt{-\frac{\pi\omega z}{2}} H_{\nu_0}^{(1)}(-\omega z) \to (-\omega z)^{\frac{1}{2}-\nu_0}, \tag{4.24}$$

and from the asymptotic side it is just (4.23). The frequency dependence can be neglected in the intermediate region. Therefore the ω-dependence of the essential combination of coefficients can be determined:

$$C_\infty^{(1)} - C_\infty^{(2)} \sim \omega^{\nu_\infty - \nu_0}. \tag{4.25}$$

4.4 Conductivity

Next we calculate the conductivity in a general $(d+2)$-dimensional spacetime. It can be seen that the asymptotic form of $\tilde{A}_x$ is

$$\tilde{A}_x = \tilde{A}_x^{(0)} + \frac{\tilde{A}_x^{(1)}}{r^{d-1}}, \tag{4.26}$$

and the conductivity takes the following form:

$$\sigma = -i\frac{d-1}{\omega}f^2(\phi_0)\frac{\tilde{A}_x^{(1)}}{\tilde{A}_x^{(0)}}, \tag{4.27}$$

where $f(\phi_0)$ denotes the asymptotic value of the gauge coupling. Therefore

$$(d-1)f^2(\phi_0)\left(\tilde{A}_x^{(1)*}\tilde{A}_x^{(0)} - \tilde{A}_x^{(1)}\tilde{A}_x^{(0)*}\right) = -2i\omega\left|\tilde{A}_x^{(0)}\right|^2 \operatorname{Re}\sigma, \tag{4.28}$$

and the asymptotic form of Ψ can be written as

$$\Psi = f(\phi_0)\left(\tilde{A}_x^{(0)}r^{\frac{d-2}{2}} + \tilde{A}_x^{(1)}r^{-\frac{d}{2}}\right). \tag{4.29}$$

Then we can obtain the conserved flux at the boundary:

$$\begin{aligned}\mathcal{F} &= i(\Psi^*\partial_z\Psi - \Psi\partial_z\Psi^*) \\ &= i(d-1)f^2(\phi_0)\left(\tilde{A}_x^{(1)*}\tilde{A}_x^{(0)} - \tilde{A}_x^{(1)}\tilde{A}_x^{(0)*}\right)\frac{1}{r^2}\frac{\partial r}{\partial z}.\end{aligned} \tag{4.30}$$

Substituting (4.28) and noting that $\partial r/\partial z = r^2$ at the boundary, we have

$$\mathcal{F} = 2\omega\left|\tilde{A}_x^{(0)}\right|^2 \operatorname{Re}\sigma. \tag{4.31}$$

Notice that $\Psi \sim r^{d/2-1}\tilde{A}_x^{(0)} \sim (-z)^{1-d/2}\tilde{A}_x^{(0)}$, then from the results (4.23) and (4.25) we have

$$\tilde{A}_x^{(0)} = -i(2)^{\nu_\infty}\left(C_\infty^{(1)} - C_\infty^{(2)}\right)\omega^{\frac{1}{2}-\nu_\infty} \sim \omega^{\frac{1}{2}-\nu_0}. \tag{4.32}$$

By evaluating the conserved flux at the horizon, we can easily check that

$$\mathcal{F} \sim \omega, \tag{4.33}$$

and thus, combining with the result (4.31), the real part of the conductivity is

$$\operatorname{Re}\sigma \sim \omega^\delta, \qquad \delta = 2\nu_0 - 1. \tag{4.34}$$

The exponent δ has the same expression as the $d = 2$ case in [9], but the value of ν_0 generically depends on the spacetime dimension and also on the dilaton coupling α.

5 Gauss-Bonnet corrections to η/s at finite temperature

In this section we will discuss the Gauss-Bonnet corrections to η/s at finite temperature in five dimensions. One remarkable progress in the AdS/CFT correspondence is the calculation of the ratio of shear viscosity over the entropy density in the dual gravity side. It has been found that $\eta/s = 1/4\pi$ for a large class of CFTs with Einstein gravity duals in the large N limit. Therefore, it was conjectured that $1/4\pi$ is a universal lower bound for all materials, which is the so-called Kovtun-Son-Starinets (KSS) bound [20]. However, in [21, 22, 23] it was observed that in R^2 gravity such a lower bound was violated, and a new lower bound $4/25\pi$ was proposed by considering the causality of the dual field theory.

It was argued in [24] that the shear viscosity is fully determined by the effective coupling of the transverse gravitons on the horizon. This was confirmed in [25] via the scalar membrane paradigm and in [26] by calculating the on-shell action of the transverse gravitons. However, the full solutions were still used in the actual calculations. Recently, η/s with higher derivative corrections was revisited for various examples in [27]. They calculated η/s in the presence of higher order corrections by making use of the near horizon data only. It turned out that the results agreed with those obtained in the previous literature. An efficient method for computing the zero frequency limit of transport coefficients in strongly coupled field theories described holographically by higher derivative gravity theories was proposed in [29].

Here we calculate η/s for black holes in five-dimensional Gauss-Bonnet gravity. Since the charged dilaton black holes have vanishing entropy at extremality, we shall not consider the zero-temperature limit. We adopt the formalism proposed in [28], where a three-dimensional effective metric $\tilde{g}_{\mu\nu}$ was introduced and the transverse gravitons were minimally coupled to this new effective metric. The action in this new formalism can take a covariant form. Similar discussions on this issue were also presented in [30].

Consider a tensor perturbation $h_{xy} = h_{xy}(t, u, z)$, where u is the radial coordinate in which the horizon is located at $u = 1$, and the momentum of the perturbation points along the z-axis. If the transverse gravitons can be decoupled from other perturbations,

the effective bulk action of the transverse gravitons can be written in a general form:

$$S = \frac{V_{x,y}}{16\pi G_5}\left(-\frac{1}{2}\right)\int d^3x\sqrt{-\tilde{g}}\left[\tilde{K}(u)\tilde{g}^{MN}\tilde{\nabla}_M\tilde{\phi}\tilde{\nabla}_N\tilde{\phi} + m^2\tilde{\phi}^2\right], \tag{5.1}$$

up to some total derivatives, where $\tilde{\phi} = h_y^x$ can be expanded as $\tilde{\phi}(t,u,z) = \tilde{\phi}(u)\mathrm{e}^{-i\omega t+ipz}$. Here $\tilde{g}_{MN}, M, N = t, u, z$ is a three-dimensional effective metric, m is an effective mass and $\tilde{\nabla}_M$ is the covariant derivative using $\tilde{g}_{MN}$. Notice that $\tilde{\phi}$ is a scalar in the three dimensions t, u, z, while it is not a scalar in the whole five dimensions. The three-dimensional effective action itself is general covariant, and $\tilde{K}(u)$ is a scalar under general coordinate transformations. In the following we will use $g_{\mu\nu}$ to denote the whole five-dimensional background.

The action of the transverse gravitons in momentum space can be written explicitly as follows

$$S = \frac{V_{x,y}}{16\pi G_5}\left(-\frac{1}{2}\right)\int \frac{d\omega dp}{(2\pi)^2}du\sqrt{-\tilde{g}}\left[\tilde{K}(u)\left(\tilde{g}^{uu}\tilde{\phi}'\tilde{\phi}' + \omega^2\tilde{g}^{tt}\tilde{\phi}^2 + p^2\tilde{g}^{zz}\tilde{\phi}^2\right) + m^2\tilde{\phi}^2\right], \tag{5.2}$$

where

$$\begin{aligned}\tilde{\phi}(t,u,z) &= \int \frac{d\omega dp}{(2\pi)^2}\,\tilde{\phi}(u;k)\,\mathrm{e}^{-i\omega t+ipz},\\ k = (\omega, 0, p), &\qquad \tilde{\phi}(u;-k) = \tilde{\phi}^*(u;k),\end{aligned} \tag{5.3}$$

and the prime denotes the derivative with respect to u. Following [28], η is given by

$$\eta = \frac{1}{16\pi G_5}\left[\sqrt{\tilde{g}_{zz}}\,\tilde{K}(u)\right]_{u=1}. \tag{5.4}$$

Next, consider a general background

$$ds^2 = -g(u)(1-u)dt^2 + \frac{du^2}{h(u)(1-u)} + \frac{r_0^2}{u^\kappa}(dx^2+dy^2+dz^2), \tag{5.5}$$

where $g(u)$ and $h(u)$ are regular functions at the horizon $u = 1$ and κ is a parameter. It turns out that the effective action of the transverse gravitons can be written in the form of (5.2) with the effective three-dimensional metric

$$\tilde{g}^{uu} = \left(1 + \frac{\lambda_{\rm GB}}{2}\frac{\kappa g'_{tt}g^{uu}}{ug_{tt}}\right)g^{uu}, \tag{5.6}$$

$$\tilde{g}^{tt} = \left[1 + \frac{\lambda_{\rm GB}}{2}\left(\frac{\kappa g'^{uu}}{u} - \frac{(\kappa^2+2\kappa)g^{uu}}{u^2}\right)\right]g^{tt}, \tag{5.7}$$

$$\tilde{g}^{zz} = \left[1 + \frac{\lambda_{\rm GB}}{2}\left(\frac{g'^2_{tt}g^{uu}}{g^2_{tt}} - \frac{g'_{tt}g'^{uu}}{g_{tt}} - \frac{2g^{uu}g''_{tt}}{g_{tt}}\right)\right]g^{zz}. \tag{5.8}$$

In fact, the effective action of the transverse gravitons can also be written as

$$S = \frac{1}{16\pi G_5}\left(-\frac{1}{2}\right)\int d^5x\sqrt{-g}\,\hat{g}^{\mu\nu}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi}, \tag{5.9}$$

where the new metric integrated Gauss-Bonnet correction is given by $\hat{g}^{\mu\nu} = \tilde{g}^{\mu\nu}$ for $\mu,\nu = t,u,z$ and $\hat{g}^{\mu\nu} = g^{\mu\nu}$ for $\mu,\nu = x,y$. Then the coupling can be computed by $\tilde{K}(u) = \sqrt{-g}/\sqrt{-\tilde{g}}$. After a straightforward calculation one can finally derive the following expression:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{\kappa}{2}\lambda_{\rm GB}h(1)\right], \tag{5.10}$$

where we have used the fact that the Bekenstein-Hawking formula still holds in Gauss-Bonnet gravity. Notice that in order to obtain corrections to η/s at the leading order of $\lambda_{\rm GB}$, it is sufficient to work in the original background (5.5).

Recall the five-dimensional black hole solution

$$ds^2 = -a^2(w)f(w)dt^2 + \frac{dw^2}{a^2(w)f(w)} + b^2(w)(dx^2+dy^2+dz^2), \quad f(w) = 1 - \frac{w_0^{3\beta+1}}{w^{3\beta+1}}, \tag{5.11}$$

we can take the following coordinate transformation:

$$\left(\frac{w_0}{w}\right)^{3\beta+1} = u^2, \tag{5.12}$$

to convert the black hole metric into the form of (5.5) with

$$\begin{aligned} &r_0 = \omega_0^\beta, \qquad \kappa = \frac{4\beta}{3\beta+1}, \\ &g(u) = -a_0^2 w_0^2 u^{-\frac{4}{3\beta+1}}(1+u), \qquad h(u) = \frac{(3\beta+1)^2}{4}a_0^2u^2(1+u). \end{aligned} \tag{5.13}$$

Now substituting all the relevant data into (5.10), we can arrive at

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{12\beta}{2\beta+1}\lambda_{\rm GB}\right). \tag{5.14}$$

Notice that $\beta \to 1$, that is, in the relativistic limit, it reproduces the well-known result obtained in [22].

It has been verified that in certain charged black hole backgrounds, the charge parameter q_e also contributes to the corrections to η/s [31, 32, 33, 34, 35, 36]. However, it seems that our result does not have any dependence on q_e. This may be understood as follows: in [27] the near horizon configuration for charged black holes contained the charge parameter

q_e, while here in the near horizon metric, the charge parameter q_e is fixed only by the parameter α after choosing $b_0 = 1$. Then by restoring the explicit dependence of b_0 in the metric, the near horizon data should contain the charge parameter q_e. Therefore one can expect that the explicit q_e dependence in the corrections to η/s might be recovered.

6 Summary and discussion

In this paper we study general $(d+2)$-dimensional charged dilaton black hole with planar symmetry obtained in [14], generalizing the investigations in [9]. Rather than treating these black holes as global solutions, here we consider them to be the near horizon solutions of a generic black hole with AdS_{d+2} asymptotic geometry. We discuss the thermodynamics of the near-extremal black holes, and we calculate the AC conductivity in the zero-temperature background. We find that the AC conductivity behaves as ω^δ, where δ is a constant determined by the parameter α in the gauge coupling and d. When $d = 2$, we reproduce the result obtained in [9]. We also calculate the Gauss-Bonnet corrections to η/s in a five-dimensional finite-temperature background. The result reduces to the previously known result in the relativistic limit. However, unlike other works studying the higher order corrections to η/s for charged black holes, our result does not depend on the charge parameter q_e. This may be due to the fact that the charge parameter is fixed by α and d after choosing a specific value for b_0, thus the near horizon configuration does not contain information about q_e explicitly. The q_e dependence of the corrections to η/s might be recovered by restoring the explicit dependence of b_0 in the metric.

One further generalization is to discuss the case of a dyonic black hole, which carries both electric and magnetic charges. One can expect that such solutions possess Lifshitz-like near horizon geometry and an AdS_{d+2} asymptotic geometry. It would be interesting to study the thermodynamics and transport coefficients, such as the Hall conductivity [37], in the presence of the magnetic field.

There have been several interesting papers investigating non-Fermi liquid states in an RN-AdS black hole background [38, 39, 40, 41]. The asymptotic geometry is AdS_{d+2} and the near horizon geometry contains an AdS_2 part, which plays a central role in the investigations. It would be worthwhile to generalize their considerations to the solutions discussed here. Note that now we have a Lifshitz-like near horizon geometry instead, and

in principle we can still calculate the corresponding correlation functions by making use of the matching technique. We expect to study such fascinating topics in the future.

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A Solving the Schrödinger equation

The Schrödinger equation with a z^{-2} potential

$$-\Psi'' + V(z)\Psi = \omega^2\Psi, \qquad V(z) = \frac{c}{z^2}, \tag{A.1}$$

can be transformed, by introducing a new variable (the range of z is $-\infty < z < 0$)

$$\Psi(z) = \chi_0\sqrt{-\omega z}\,\chi(z), \tag{A.2}$$

to the Bessel equation:

$$z^2\partial_z^2\chi + z\partial_z\chi + (\omega^2 z^2 - \nu^2)\chi = 0, \qquad \nu^2 = c + \frac{1}{4}. \tag{A.3}$$

The solutions are the Hankel functions

$$\chi(z) = C_1 H_\nu^{(1)}(-\omega z) + C_2 H_\nu^{(2)}(-\omega z). \tag{A.4}$$

The approximative formulae for the Hankel functions are [42]

$$H_\nu^{(1)}(-\omega z) \;\to\; -i\frac{\Gamma(\nu)}{\pi}\left(\frac{-\omega z}{2}\right)^{-\nu}, \qquad -\omega z \to 0, \tag{A.5}$$

$$H_\nu^{(2)}(-\omega z) \;\to\; i\frac{\Gamma(\nu)}{\pi}\left(\frac{-\omega z}{2}\right)^{-\nu}, \qquad -\omega z \to 0, \tag{A.6}$$

$$H_\nu^{(1)}(-\omega z) \;\sim\; \sqrt{-\frac{2}{\pi\omega z}}\,\mathrm{e}^{-i(\omega z + \frac{1}{2}\nu\pi + \frac{1}{2}\pi)}, \qquad -\omega z \sim \infty, \tag{A.7}$$

$$H_\nu^{(2)}(-\omega z) \;\sim\; \sqrt{-\frac{2}{\pi\omega z}}\,\mathrm{e}^{i(\omega z + \frac{1}{2}\nu\pi + \frac{1}{2}\pi)}, \qquad -\omega z \sim \infty. \tag{A.8}$$

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